647510 **MSC2MatE201x** **Seat No :________**

M.Sc. (Maths) Semester - 2 (*CBCS*) Examination
May/June-2022 (NEW COURSE)
Classical Mechanics – 2 (Elective)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q-1 (A) **Answer any Two out of Three** **[10]**

(1) In usual notations derive $T = I\omega^2$.

(2) For the torque free motion of a rigid body, obtain Euler's equations of motion.

(3) Obtain Lagrangian for a heavy symmetrical top with one point fixed.

Q-1 (B) **Answer any One out of Two** **[4]**

(1) Giving all details define inertia tensor.

(2) Find the principal moments of inertia about the center of mass of flat rigid body in the shape of a 45° right triangle with uniform mass density. What are the principal axes?

Q-2 (A) **Answer any Two out of Three** **[10]**

(1) Derive the relativistic law of addition of velocities.

(2) In usual notations derive $E = mc^2$

(3) Define velocity 4-vector and find its norm.

Q-2 (B) **Answer any One out of Two** **[4]**

(1) Describe postulates of special relativity.

(2) Show that 4-dimensional volume element is invariant under special Lorentz transformation.

Q-3 (A) **Answer any Two out of Three** **[10]**

(1) Derive Lagrange's equations of motion from Hamilton's equations of motion.

(2) Obtain Hamilton's equations of motion for simple pendulum using Lagrangian $L = \frac{m}{2} l^2\dot{\theta}^2 + mgl\cos\theta$.

(3) Obtain Routhian equations of motion for a system with Lagrangian

$$L = \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{k}{r},$$

where m and k are constants.

Q-3 (B) **Answer any One out of Two** **[4]**

(1) In usual notations, derive Hamiltonian equations of motion.

(2) Find Lagrangian corresponding to the Hamiltonian $H = \frac{1}{2m}(p^2 + m^2\omega^2 q^2)$, the notations being usual.

Q-4 (A) **Answer any Two out of Three** **[10]**

(1) Solve the problem of simple harmonic oscillator using Hamilton-Jacobi theory.

(2) In usual notations, prove that

(i) $[au + bv, w] = a[u, w] + b[v, w]$,

(ii) $[uv, w] = u[v, w] + v[u, w]$,

where a, b are constants.

(3) State and prove Jacobi's identity for Poisson brackets.

Q-4 (B) **Answer any One out of Two** **[4]**

(1) Evaluate Poisson bracket $[u, v]_{q,p}$ for $u = p_1 - 2p_2$ and $v = -2q_1 - q_2$

(2) Show that if u and v are constants of motion then $[u, v]$ is also a constant of motion.

Q-5 (A) **Answer any Two out of Three** **[10]**

(1) Describe Eulerian angles.

(2) For the Hamiltonian $H = \frac{1}{2m}\left(p_x^2 + p_y^2\right) - mgy$, obtain all conserved quantities.

(3) Discuss principle of least action in Hamiltonian formalism.

Q-5 (B) **Answer any One out of Two** **[4]**

(1) Giving all the necessary details describe Routhian procedure.

(2) For fundamental Poisson's brackets prove that

(i) $\left[q_j, q_k\right]_{q,p} = 0$

(ii) $\left[q_j, p_k\right]_{q,p} = \delta_{jk}$.

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